Stakeholder Optimization Impacts on Utility Planning and Pricing

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1. Introduction

Under a traditional regulatory model that provides utilities with an opportunity to earn a rate-of-return, past research has identified that utilities have an incentive to add investment as a primary way to grow earnings[1]. This has been the underlying utility business model since the inception of rate-base regulation. At the same time, other stakeholders in regulatory proceedings possess different objectives that drive their positions. Other typical stakeholders may include the regulatory body, consumer advocates, industrial groups, and other advocates such as those with a focus on low income consumers or environmental issues. This mix of competing positions and objectives substantially increase the difficulty facing regulatory bodies as they sort through positions in order to arrive at "balanced" decisions. One usually concludes that the general regulatory objective involves maximizing societal welfare while preserving the financial health of the utility. On the one hand, the utility is likely interested in maximizing profits. On the other hand, customer-focused advocates may want to minimize revenue requirements, though still preserving the financial viability of the utility. And further, many other stakeholders are looking to maximize consumer surplus or, for an environment or energy efficiency advocate, to minimize the consumption of energy.

The pursuit of these competing objectives can impact the utility business model. The economics literature contains numerous articles on the implications of underlying objectives of utilities in a profit-maximizing mode vs. a welfare maximizing mode. The Averch-Johnson-Wellisz articles[2] are known for conclusions regarding the incentive for utilities to invest under rate of return regulation.

In contrast, articles on peak-load pricing[3] generally cover the pricing implications under a welfare maximization objective. However, in utility rate proceedings, it can be important for the

[1] Harvey Averch and Leland Johnson, "Behavior of the Firm Under Regulatory Constraint," American Economic Review, 52 (December, 1962), pp. 1052-69; and Stanislaw H. Wellisz, "Regulation of Natural Gas Pipeline Companies: An Economic Analysis" Journal of Political Economy, 71 (February, 1963), pp. 30-43.
[2] Ibid., Averch and Johnson (1962) and Wellisz (1963).
[3] E.E. Bailey and L.J. White, "Reversals in Peak and Off-Peak Pricing," The Bell Journal of Economics and Management Science, 5(Spring, 1974), pp. 75-92; Michael Crew and Paul Kleindorfer, "Marshall and Turvey on Peak-load or Joint Product Pricing," Journal of Political Economy, 79 (November, 1971), pp.

regulatory commission to understand the resulting implications from pursuit of the goals and objectives of various stakeholders. In addition, utilities and other stakeholders can arrive at a settlement agreement to resolve all issues through negotiation rather than relying solely on a Commission decision from a litigated hearing. In that case, the settlement agreement is presented to the Commission as a take-it-or-leave-it resolution for its consideration as opposed to a litigated hearing. The Commission may reject the settlement agreement or even modify it.

The give and take inherent in a settlement negotiation process may result in an acceptable or "reasonable" conclusion, but the impacts on economic efficiency may be less clear. This is also true in litigated proceedings when decision makers try to evaluate the implications of competing positions from the various stakeholders. In such cases, these regulatory processes can result in sub-optimal outcomes unless there is effort to understand the full implications on major stakeholders. Decisions to adopt alternative positions may be justified due to policy and preference considerations, but the economic efficiency implications should at least be acknowledged in the decision making process.

This paper provides insights on the welfare and utility profit implications assuming competing optimization goals across multiple stakeholders in the regulatory process. In the next section, two general models are briefly reviewed: one for investment decisions and the other for pricing decisions. Then, in the following sections, the optimization results related to different stakeholder positions are provided for each model type with the implications on investment and pricing summarized. This leads to a final discussion on potential directions for future utility business models. This leads to conclusions on pricing strategy for utilities as well as insights for utility commissions in evaluating utility proposals.

2. General Models

Investment

In late 1962 and early 1963, separate articles by Averch and Johnson[4] and S.H. Wellisz[5] (A-J-W) were published that identified that rate-of-return on rate base regulation provided an incentive for utilities to over-invest in assets. Their research pointed out that when the rate of return is above the cost of capital, this effectively lowers the price of capital and hence encourages utilities to add more capital inputs relative to labor.

Below is a brief synopsis of the A-J-W methodology under a profit maximization objective for the utility.

1369-77; Michael Crew and Paul Kleindorfer, "Peak Load Pricing with a Diverse Technology," The Bell Journal of Economics, 7 (Spring, 1976), p. 207; Ralph Turvey, "Peak-Load Pricing," Journal of Political Economy, 76 (January, 1968), pp. 101-113; and Oliver Williamson, "Peak-load Pricing and Optimal capacity under Indivisibility Constraints," American Economic Review, 56 (September, 1966), pp. 810-827.

[4] Ibid., Averch-Johnson (1962).

[5] Ibid., Wellisz (1963).

Let:

K	= capital,
L	= labor,
Q	= f(K,L) where Q is output,
P	= an inverse demand function P(Q(K,L)) where P is price,
r	= cost of capital,
s	= allowed rate of return on capital, and
w	= wage rate
MP(K)	= marginal product of capital

As indicated by A-J-W, if $s < r$, the utility will not invest, and the issue of investment becomes a moot point since the utility will not continue to operate if it cannot cover its costs. So, if $s > r$, the problem becomes relevant as to whether or not the utility has an incentive to over-invest in capital.

Using the A-J-W model, the profit maximization optimization problem can be expressed as follows:

Maximize Profit (View of the Utility)

$$\text{s.t.} \quad P(Q)\cdot Q - s\cdot K - w\cdot L = 0$$

The associated Lagrangian is:

(1) $\text{Max } L = P(Q)\cdot Q - r\cdot K - w\cdot L - \lambda 2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$

where $\lambda 2$ represents the Lagrange multiplier on the revenue/cost constraint.

The A-J-W model examines the tradeoff between capital and labor, i.e., the slope of the isoquant. The expected relationship for the isoquant is:

(2) $-dL/dK = r/w$

However, A-J-W found the following:

(3) $-dL/dK = (r - (\lambda 2\cdot(s - r))/(1 - \lambda 2))/w$ which implies that $-dL/dK < r/w$

With $r = r - (\lambda 2\cdot(s - r))/(1 - \lambda 2)$, we see that under rate base regulation with a utility working to maximize profit, the relative cost of r exceeds the marginal value of the product from capital by $(\lambda 2\cdot(s - r))/(1 - \lambda 2)$, which implies that the utility has increased the use of capital in its investment decisions. This led to the A-J-W conclusion that utilities would have an incentive to over-invest in capital, thus taking the utility off an efficient expansion path.

<u>Pricing</u>

A significant amount of literature has been published on utility pricing about price discrimination, peak / off-peak pricing, time-of-use prices, and real time pricing. All of these are important topics, but the focal point here is on the implications for pricing if the Commission were to follow the objective of one of the non-utility stakeholders. One key consideration must be included and that is the constraint on the need for the utility to have installed enough capacity to meet peak demands. In that context, the model employed here follows the structure employed by Turvey, Williamson, and Crew and Kleindorfer[6] and others to identify optimal pricing given a constraint that the utility has installed enough capacity to meet peak demands while including the constraint that revenues cover costs.

Under a peak load pricing structure (PLP), let:

Q1 = off-peak load
P1 = off-peak price
Q2 = peak load
P2 = peak price
C(Q1) = operating cost during off-peak periods
MCQ1 = off-peak marginal operating cost
C(Q2) = operating cost during peak periods
MCQ2 = peak marginal operating cost
K = capacity or capital required for a level of capacity
r = cost of capital (including depreciation)
λ1, λ2 = Lagrange multipliers

The welfare optimization problem can be expressed as:

Maximize Welfare (View of a Utility Commission)
s.t.
K – Q2 >= 0 and
P1·Q1 + P2·Q2 – C(Q1) – C(Q2) – r·K = 0

Defining consumer value of service (VOS) as:

$$(4)\quad VOS = \int_0^{Q1(0)} f(P1)dQ1 + \int_0^{Q2(0)} f(P2)dQ2$$

where P(Q1) and P(Q2) are the inverse demand functions. The problem becomes:

Max Welfare (Consumer and Producer Surplus) =

$$\int_0^{Q1(0)} f(P1)dQ1 + \int_0^{Q2(0)} f(P2)dQ2 - C(Q1) - C(Q2) - r\cdot K$$

s.t. Q2 <= K

P1·Q1 + P2·Q2 – C(Q1) – C(Q2) – r·K = 0

[6] Ibid, Turvey (1968), Williamson (1966), and Crew and Kleindorfer (1971).

Lagrangian problem is:

(5) $\text{Max } W = VOS - C(Q1) - C(Q2) - r \cdot K + \lambda 1(K-Q2) + \lambda 2(P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K)$

From this structure, we find the typical conditions:

- $P1 = MCQ1$
- $P2 = MCQ2 + (\lambda 1/(1+\lambda 2))$
- $K = Q2$
- $r = (\lambda 1/(1+\lambda 2)$

which makes $P2 = MCQ2 + r$ or marginal operating cost plus marginal cost of capital. These are the typical standard conclusions for welfare maximization. That is, off-peak prices should cover marginal operating costs, while on-peak prices should cover operating and capital costs.

A comparison of these two model structures (investment under A-J-W and pricing to maximize welfare) sets the stage for an investigation into the results under alternate stakeholder objectives.

3. Stakeholder Investment Models

The A-J-W model examined the investment decision with a utility objective to maximize profit. Alternate objectives that could apply include:

A. Maximize profit, but add a capacity constraint (utility perspective)
B. Maximize welfare (utility commission perspective)
C. Maximize welfare, but add a capacity constraint (utility commission perspective)
D. Maximize the value of service (consumer perspective)
E. Maximize the value of service, but add a capacity constraint (consumer perspective)
F. Minimize sales (environmental perspective)
G. Minimize sales but add a capacity constraint (environmental perspective)
H. Minimize costs (consumer advocate perspective)
I. Minimize costs but add a capacity constraint (consumer advocate perspective)

Examining the addition of a capacity constraint (Case A) seems a natural extension of the basic A-J-W model. Given that the utility must invest in enough capacity to meet the peak demand, there could be an additional incentive for the utility to over-invest.

By adding a capacity constraint, the A-J-W model changes to the following:

Maximize Profit with a Capacity Constraint (alternate view of the utility)

s.t. $Q \leq K$

$P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian may be expressed as:

(6) $\text{Max } L = P(Q)\cdot Q - r\cdot K - w\cdot L - \lambda1\cdot(Q - K) - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$

Solving the optimization problem for r, we find that:

(7) $-dL/dK = (r - (\lambda1 + \lambda2\cdot(s - r))/(1 - \lambda2))/w$

where $(\lambda1 + \lambda2\cdot(s - r))/(1 - \lambda2) > 0$ where $0 < \lambda2 < 1$

In this situation, with $r = r - (\lambda1 + \lambda2\cdot(s - r))/(1 - \lambda2)$, we see that under regulation and a utility working to maximize profit, the relative cost of r exceeds the marginal value of the product from capital by $(\lambda1 + \lambda2\cdot(s - r))/(1 - \lambda)$, which implies that the utility has a greater incentive to increase the use of capital in its investment decisions . This further accentuates the A-J-W conclusion that utilities would have a greater incentive to over-invest in capital and move off the efficient expansion path, because there is also the pressure to have enough capacity to meet consumer demand.

Using the same general model, as Tables 1 and 2 below show, Cases B through I can be examined to define the implications on investment under alternate stakeholder objectives, with and without the inclusion of a capacity constraint. The respective Lagrangian optimization problems for the investment focused model are summarized in Table1 as follows:

Table 1: Investment Optimization Cases

Case	Objective	Lagrangian
Original Model	Maximize Profit	$\text{Max } L = P(Q)\cdot Q - r\cdot K - w\cdot L - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
A	Maximize Profit with Capacity Constraint	$\text{Max } L = P(Q)\cdot Q - r\cdot K - w\cdot L - \lambda1\cdot(Q - K) - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
B	Maximize Welfare	$\text{Max } W = \int_0^Q P(Q)\,dQ - r\cdot K - w\cdot L - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
C	Maximize Welfare with Capacity Constraint	$\text{Max } W = \int_0^Q P(Q)\,dQ - r\cdot K - w\cdot L - \lambda1\cdot(Q - K) - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
D	Maximize the value of service	$\text{Max } W = \int_0^Q P(Q)\,dQ - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
E	Maximize the value of service with Capacity Constraint	$\text{Max } W = \int_0^Q P(Q)\,dQ - \lambda1\cdot(Q - K) - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
F	Minimize Sales	$\text{Min } L = Q - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
G	Minimize Sales with Capacity Constraint	$\text{Min } L = Q - \lambda1\cdot(Q - K) - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
H	Minimize Costs	$\text{Min } L = r\cdot K + w\cdot L - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$
I	Minimize Costs with Capacity Constraint	$\text{Min } L = r\cdot K + w\cdot L - \lambda1\cdot(Q - K) - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$

The key differences in the cases are about the objective function. From the first-order equations for each of the models, we find the following relationships as summarized in Table 2. Appendix A provides the derivations for these results.

Table 2: Investment Optimization Results				
Case	Objective	Isoquant	Is $-dL / dK$ > or < r/w?	Incentive to Invest
Original Model	Maximize Profit	$(r-(\lambda 2 \cdot (s-r))/(1-\lambda 2))/w$	$-dL / dK < r/w$	Over
A	Maximize Profit with Capacity Constraint	$(r-(\lambda 1+\lambda 2 \cdot (s-r))/(1-\lambda 2))/w$	$-dL / dK < r/w$	Over
B	Maximize Welfare	$(r-(\lambda 2 \cdot (s-r))/(1-\lambda 2))/w$	$-dL / dK < r/w$	Over
C	Maximize Welfare with Capacity Constraint	$(r-(\lambda 1+\lambda 2 \cdot (s-r))/(1-\lambda 2))/w$	$-dL / dK < r/w$	Over
D	Maximize the value of service	$(r+(r+\lambda 2 \cdot (s-r))/(\lambda 2))/w$	$-dL / dK > r/w$	Under
E	Maximize the value of service with Capacity Constraint	$(r+(r+\lambda 1+\lambda 2 \cdot (s-r))/(\lambda 2))/w$	$-dL / dK > r/w$	Under
F	Minimize Sales	$(r+(MP(K)+\lambda 2 \cdot (s-r))/(\lambda 2))/w$	$-dL / dK > r/w$	Under
G	Minimize Sales with Capacity Constraint	$(r+(MP(K)+\lambda 1+\lambda 2 \cdot (s-r))/(\lambda 2))/w$	$-dL / dK > r/w$	Under
H	Minimize Costs	$(r+(r+\lambda 2 \cdot (s-r))/(\lambda 2))/w$	$-dL / dK > r/w$	Under
I	Minimize Costs with Capacity Constraint	$(r+(r+\lambda 1+\lambda 2 \cdot (s-r))/(\lambda 2))/w$	$-dL / dK > r/w$	Under

For objectives to maximize profit or welfare, the relative cost of r exceeds the marginal value of the product from capital which implies that the utility has a greater incentive to increase the use of capital in its investment decisions. This occurs whether or not the model includes the capacity constraint, though inclusion of the capacity constraint increases the incentive to over-invest. For the other objectives, where the utility is to minimize sales, minimize costs, and maximize VOS, the relative cost of r is below the marginal value of the product from capital which implies that the utility would not have an incentive to increase the use of capital in its investment decisions. This also is the result whether or not the model includes the capacity constraint. The results for these other three objectives are fairly similar. In fact the objective to maximize customer VOS produces the same conclusion as the objective to minimize costs.[7]

In regulatory proceedings, such as those associated with a utility's integrated resource plan (IRP) or application for approval to build a new facility, this issue of over or under investing in new plant deserves some attention. If the utility commission's objective is to maximize welfare, the implication of this objective oddly matches the profit maximization objective of the utility. At the same time, the objective of the utility commission may conflict with those stakeholders that have an objective to minimize sales, minimize costs, or maximize VOS. It may be that the desire of the Commission to ensure adequate capacity to meet consumer demands aligns more with the utility profit objective, but as a result ends up in opposition to other stakeholders.

To obtain additional insights on this issue, we now turn to the pricing side.

4. Stakeholder Pricing Models

As previously noted, the pricing model research has tended to focus on the benefit of peak and off-peak pricing for the purpose of achieving an efficient allocation of resources. Average cost pricing, usually the result of utility rate proceedings, tends to encourage an increase in demand during peak periods, thus requiring utilities to invest in additional plant in order to meet those peak demands. Instead of delving into the much researched area of peak load pricing, we are using the peak / off-peak pricing structure as a proxy for the level of pricing sought by different stakeholders. With respect to the original pricing

[7] If one were to also examine the objective to minimize customer bills as defined by total revenues (price times quantity), the results are the same as those found when minimizing costs or maximizing the VOS.

model discussed which relies on a welfare maximization objective, alternate objectives that could also apply are:

J. Maximize Profit (utility perspective)
K. Maximize Value of Service (consumer perspective)
L. Minimize sales (environmental perspective)
M. Minimize costs (consumer advocate perspective)

Using the same general model structure as demonstrated in equation (5), Cases J through M can be examined to assess the pricing implications with respect to alternate stakeholder objectives. These Lagrangian optimization problems for the pricing focused model are summarized in Table3 as follows:

Table 3: Pricing Optimization Cases

Case	Objective	Lagrangian
Original Model	Maximize Welfare	$\int_0^{Q1(0)} f(P1)dQ1 + \int_0^{Q2(0)} f(P2)dQ2$ - C(Q1) – C(Q2) – r· K + λ1(K-Q2) + λ2(P1· Q1 + P2 · Q2 – C(Q1) – C(Q2) – r · K)
J	Maximize Profit	Max L = P1· Q1 + P2· Q2 –C(Q1) – C(Q2) – r· K + λ1(K-Q2) + λ2(P1· Q1 + P2· Q2 – C(Q1) – C(Q2) – r· K)
K	Maximize Value of Service	$\int_0^{Q1(0)} f(P1)dQ1 + \int_0^{Q2(0)} f(P2)dQ2$ + λ1(K-Q2) + λ2(P1· Q1 + P2 · Q2 – C(Q1) – C(Q2) – r · K)
L	Minimize Sales	Min L = Q1 + Q2 + λ1(K-Q2) + λ2(P1· Q1 + P2· Q2 – C(Q1) – C(Q2) – r· K)
M	Minimize Costs	Min L = C(Q1) +C(Q2) + r· K + λ1(K-Q2) + λ2(P1· Q1 + P2· Q2 – C(Q1) – C(Q2) – r· K)

Again, there are key differences in the respective objective functions. From the first-order equations for each of the models, we find the following relationships as summarized in Table 4. Appendix B provides the derivations for these results. For the objectives to maximize profit or welfare, the pricing results are the same. This implies that as we found with the

Table 4: Pricing Optimization Results

Case	Objective	Off-Peak Pricing	Peak Pricing	Comparison to Welfare Max Price
Original Model	Maximize Welfare	P1 = MCQ1	P2 = MCQ2 + r	Equal
J	Maximize Profit	P1 = MCQ1	P2 = MCQ2 + r	Equal
K	Maximize Value of Service	P1 = MCQ1· (λ1/(r+λ1))	P2 = (MCQ2 + r)· (λ1/(r+λ1))	Lower
L	Minimize Sales	P1 = MCQ1 – r/λ1	P2 = MCQ2 + r - r/λ1	Lower
M	Minimize Costs	P1 = MCQ1· (λ1/(r+λ1))	P2 = (MCQ2 + r)· (λ1/(r+λ1))	Lower

investment model, the objective of the utility under the pricing model produces results that align with the welfare maximization objective of the utility commission. Conversely, the pricing results from pursuit of other objectives (maximize VOS, minimize sales, or minimize costs) imply that prices would be set below those found if the welfare maximization objective is pursued. As we found for the investment model, Cases K and M (maximize VOS and minimize costs) produce the same results[8]. While Case L also produces lower pricing results, it is interesting to note that under Case L, minimizing sales, the lower prices would actually encourage increases in consumer demand. When coupling with the result that minimizing the sales objective encourages under-investment in plant, this points to an inherent

[8] And this also applies to minimizing customer bills as defined using total revenues.

inconsistency with the pursuit of a sales minimization objective. As with the investment model, the objectives of the commission may not align with other non-utility stakeholders. Conclusions and Implications for Business Models

While a utility commission has multiple, often competing, objectives to consider when it decides, it is useful to more fully understand the implications of alternate stakeholder positions on pricing and investment. If one examines competing positions set forth in a regulatory proceeding or one evaluates the results of a negotiated settlement of all parties, this analysis suggests the need for a commission to further consider how to ensure that its decisions promote the efficient allocation of resources.

Now, as we know, utilities generally charge average cost prices, not marginal cost prices as utilized in this paper. A useful extension of this analysis would be to examine the pricing implications under alternate stakeholder objectives when the utility charges average costs. Differences between average cost prices and marginal cost prices create opportunities for pricing arbitrage and cherry-picking of customer loads through the installation of distributed energy resources.

The implications for business models is that utilities will be forced to migrate from average cost pricing and instead pursue new pricing approaches to minimize the ability of other suppliers to price arbitrage and/or use new technologies to cherry-pick customer loads.

The methodology development in this paper can be extended to understand the implications on investment and pricing as the utility industry incorporates the impacts of distributed energy resources, including renewable resources as well as energy efficiency and demand response programs.

References:

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7. Williamson, Oliver. "Peak-load Pricing and Optimal capacity under Indivisibility Constraints," American Economic Review, 56 (September, 1966), pp. 810-827.

Appendix A: Investment Model

Using the model structure in the Averch-Johnson paper, the following analyses examine how this result varies under alternate objective functions.

Assume:

K = capital
L = labor
$Q = f(K,L)$
P = an inverse demand function $P(Q(K,L))$
r = cost of capital
s = allowed rate of return on capital
w = wage rate

As indicated by AJW, if $s < r$, the utility will not invest, and the issue is a moot point since the utility will not operate

So, if $s > r$, the problem becomes more interesting in terms of identifying whether or not the utility has an incentive to over-invest.

Using the AJW model, the optimization problem can be expressed as:

1. Max Profit (View of the Utility)

s.t. $P(Q)\cdot Q - s\cdot K - w\cdot L = 0$

The Lagrangian is:

$$\text{Max } L = P(Q)\cdot Q - r\cdot K - w\cdot L - \lambda2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$$

Then, taking derivatives, Kuhn-Tucker equations become:

$$LK = (P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial K) - r - \lambda2\cdot(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial K) + \lambda2\cdot s = 0$$

$$LL = (P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial L) - w - \lambda2\cdot(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial L) + \lambda2\cdot w = 0$$

$$L2 = P(Q)\cdot Q - s\cdot K - w\cdot L$$

Now, as Averch-Johnson pointed out, the marginal value of the product from K or L will be equal to the marginal cost of the input (r or w). Therefore:

$$(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial K) = r \quad \text{and}$$

$$(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial L) = w$$

Substituting and rearranging terms produces:

(a) $r - r - \lambda2\cdot r + \lambda2\cdot s = 0$ or $r\cdot(1 - \lambda2) - r + \lambda2\cdot s = 0$
(b) $(1 - \lambda2)\cdot w - w + \lambda2\cdot w = 0$

(c) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a), we find:

(a) $r = r - (\lambda 2 \cdot (s - r))/(1 - \lambda 2)$ where $(\lambda 2 \cdot (s - r))/(1 - \lambda 2) > 0$ where $0 < \lambda 2 < 1$

This is derived as follows:

$r = (r - \lambda 2 \cdot s)/(1 - \lambda 2)$

$= (r - \lambda 2 \cdot s + \lambda 2 \cdot r - \lambda 2 \cdot r)/(1 - \lambda 2)$

$= (r \cdot (1 - \lambda 2) - \lambda 2 \cdot (s - r))/ (1 - \lambda 2)$ which becomes (a) above.

Now, looking at the tradeoff between capital and labor, the slope of the isoquant would be:

$-dL/dK = r/w = (r - (\lambda 2 \cdot (s - r))/(1 - \lambda 2))/w$ which implies that $-dL/dK < r/w$

With $r = r - (\lambda 2 \cdot (s - r))/(1 - \lambda 2)$, we see that under regulation and a utility working to maximize profit, the relative cost of r exceeds the marginal value of the product from capital by $(\lambda 2 \cdot (s - r))/(1 - \lambda 2)$, which implies that the utility has increased the use of capital in its investment decisions. This led to the A-J-W conclusion that utilities would have an incentive to over-invest in capital which takes the utility off the efficient expansion path.

Before turning to alternate objective functions, the AJW model overlooks one other constraint, namely, that the amount of capital investment has to at least provide the capacity to cover consumer demand. This leads to the following model specification.

2. Max Profit with a Capacity Constraint (Alternate View of the Utility)

s.t. $Q2 <= K$
$P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian may be expressed as:

$\text{Max } L = P(Q) \cdot Q - r \cdot K - w \cdot L - \lambda 1 \cdot (Q - K) - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) - r + \lambda 1 - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$

$LL = (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) - w - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$

$L\lambda 1 = K - Q = 0$

$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L$

Substituting as before and rearranging terms produces:

(a) $r - r + \lambda 1 - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$ or $r \cdot (1 - \lambda 2) - r + \lambda 1 + \lambda 2 \cdot s = 0$
(b) $(1 - \lambda 2) \cdot w - w + \lambda 2 \cdot w = 0$
(c) $K - Q = 0$
(d) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a), we find:

(a) $r = r - (\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda 2)$ where $(\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda 2) > 0$ where $0 < \lambda 2 < 1$

In this situation, with $r = r - (\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda 2)$, we see that under regulation and a utility working to maximize profit, the relative cost of r exceeds the marginal value of the product from capital by $(\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda)$, which implies that the utility has a greater increase in the use of capital in its investment decisions . This further accentuates the AJW conclusion that utilities would have a greater incentive to over-invest in capital and move off the efficient expansion path, because there is also the pressure to have enough capacity to meet consumer demand.

3. Max Welfare (View of the Commission)

Max Welfare

s.t. $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

Max $W = \int_0^Q P(Q)dQ - r \cdot K - w \cdot L - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = (P) \cdot (\partial Q/\partial K) - r - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$

$LL = (P) \cdot (\partial Q/\partial L) - w - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$

$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L$

Since $(P) \cdot (\partial Q/\partial K)$ and $(P) \cdot (\partial Q/\partial L)$ are also the respective values of the marginal product for K and L, we can set those equal to r and w.

So, substituting and rearranging terms produces:

(a) $r - r - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$ or $r \cdot (1 - \lambda 2) - r + \lambda 2 \cdot s = 0$
(b) $(1 - \lambda 2) \cdot w - w + \lambda 2 \cdot w = 0$
(c) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a), we find:

(a) $r = r - (\lambda 2 \cdot (s - r))/(1 - \lambda 2)$ where $(\lambda 2 \cdot (s - r))/(1 - \lambda 2) > 0$ where $0 < \lambda 2 < 1$

which is the same result found under a profit maximization objective function. So, both under profit maximization and welfare maximization, rate of return regulation provides the same incentive for utilities to invest in capital plant.

4. Max Welfare with a Capacity Constraint (Alternate View of the Commission)

The welfare maximum problem can also be examined like the profit maximization problem with the addition of the requirement that capacity must exceed demand. This leads to:

Max Welfare

s.t. $Q <= K$
$P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

$$\text{Max } W = \int_0^Q P(Q)dQ - r \cdot K - w \cdot L - \lambda 1 \cdot (Q - K) - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$$

Then, taking derivatives, Kuhn-Tucker equations become:

$$LK = (P) \cdot (\partial Q/\partial K) - r + \lambda 1 - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$$

$$LL = (P) \cdot (\partial Q/\partial L) - w - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$$

$$L\lambda 1 = K - Q = 0$$

$$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L$$

Since $(P) \cdot (\partial Q/\partial K)$ and $(P) \cdot (\partial Q/\partial L)$ are also the respective values of the marginal product for K and L, we can set those equal to r and w.

So, substituting and rearranging terms produces:

(a) $r - r + \lambda 1 - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$ or $r \cdot (1 - \lambda 2) - r + \lambda 2 \cdot s = 0$
(b) $(1 - \lambda 2) \cdot w - w + \lambda 2 \cdot w = 0$
(c) $K - Q = 0$
(d) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a), we find the same results as under the profit maximization case with a capacity constraint:

(a) $r = r - (\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda 2)$ where $(\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda 2) > 0$ where $0 < \lambda 2 < 1$

In this situation, with $r = r - (\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda 2)$, we again see that under rate of return regulation and a utility working to maximize profit, the relative cost of r exceeds the marginal value of the product from capital by $(\lambda 1 + \lambda 2 \cdot (s - r))/(1 - \lambda)$, which implies that the utility has a greater increase in the use of capital in its investment decisions . This further accentuates the AJW conclusion that utilities would have a greater incentive to over-invest in capital and move off the efficient expansion path, because there is also the pressure to have enough capacity to meet consumer demand.

5. Max VOS (View of the Consumer)

For this case, we are interested to see the impact on investment decisions should one try to maximize the VOS as an objective. Here, the problem can be stated as:

Max VOS

s.t. $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

Max $W = \int_0^Q P(Q)dQ - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = (P) \cdot (\partial Q/\partial K) - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$

$LL = (P) \cdot (\partial Q/\partial L) - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$

$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

Performing substitutions and rearranging terms produces:

(a) $r - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$
(b) $(1 - \lambda 2) \cdot w + \lambda 2 \cdot w = 0$
(c) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a) we find:

(a) $r = r + (r + \lambda 2 \cdot (s - r))/(\lambda 2)$ where $(r + \lambda 2 \cdot (s - r))/(\lambda 2) > 0$

Now we have a different conclusion. Instead of an incentive to over-invest in capital, pursuit of a VOS objective function leads one to under-invest in capital since the relative cost of capital is less than the marginal value of the product from capital.

6. Max VOS with a Capacity Constraint (View of the Consumer)

For this case, we are interested to see the impact on investment decisions should one try to maximize the VOS as an objective. Here, the problem can be stated as:

Max VOS

s.t. $Q <= K$
$P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

Max $W = \int_0^Q P(Q)dQ - \lambda 1 \cdot (Q - K) - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = (P) \cdot (\partial Q/\partial K) + \lambda 1 - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$

$LL = (P) \cdot (\partial Q/\partial L) - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$

$L\lambda 1 = K - Q = 0$

$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

Performing substitutions and rearranging terms produces:

(a) $r + \lambda 1 - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$
(b) $(1 - \lambda 2) \cdot w + \lambda 2 \cdot w = 0$
(c) $K - Q = 0$
(d) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a) we find:

(a) $r = r + (r + \lambda 1 + \lambda 2 \cdot (s - r))/(\lambda 2)$ where $(r + \lambda 1 + \lambda 2 \cdot (s - r))/(\lambda 2) > 0$

Here we have a similar conclusion. Instead of an incentive to over-invest in capital, pursuit of a VOS objective function leads one to under-invest in capital since the relative cost of capital is less than the marginal value of the product from capital. The incentive is even higher with the inclusion of λ1 in the solution.

7. Min Sales (View of the Environmentalist Intervener)

For this case, we are interested to see the impact on investment decisions should one try to minimize sales as an objective. Here, the problem can be stated as:

Min Q(K,L)

s.t. $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

$\text{Min } L = Q - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = (\partial Q/\partial K) - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$

$LL = (\partial Q/\partial L) - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$

$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

Let $(\partial Q/\partial K)$ and $(\partial Q/\partial L)$ be represented by MP(K) and MP(L).

Performing substitutions and rearranging terms produces:

(a) $MP(K) - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$
(b) $MP(L) - \lambda 2 \cdot w + \lambda 2 \cdot w = 0$
(c) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a) we find:

(a) $r = r + (MP(K) + \lambda 2 \cdot (s - r))/(\lambda 2)$ where $(MP(K) + \lambda 2 \cdot (s - r))/(\lambda 2) > 0$

Again, instead of an incentive to over-invest in capital, pursuit of a minimization of sales objective function leads one to under-invest in capital since the relative cost of capital is less than the marginal value of the product from capital.

8. Min Sales with a Capacity Constraint (View of the Environmentalist Intervener)

For this case, we are interested to see the impact on investment decisions should one try to minimize sales as an objective while having adequate capacity to meet demand. Here, the problem can be stated as:

$\text{Min } Q(K,L)$

s.t. $Q \le K$

$P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

$\text{Min } L = Q - \lambda 1 \cdot (Q - K) - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = (\partial Q/\partial K) + \lambda 1 - \lambda 2\cdot(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial K) + \lambda 2\cdot s = 0$

$LL = (\partial Q/\partial L) - \lambda 2\cdot(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial L) + \lambda 2\cdot w = 0$

$L\lambda 1 = K - Q = 0$

$L\lambda 2 = P(Q)\cdot Q - s\cdot K - w\cdot L = 0$

Let $(\partial Q/\partial K)$ and $(\partial Q/\partial L)$ be represented by MP(K) and MP(L).

Performing substitutions and rearranging terms produces:

(a) $MP(K) + \lambda 1 - \lambda 2\cdot r + \lambda 2\cdot s = 0$
(b) $MP(L) - \lambda 2\cdot w + \lambda 2\cdot w = 0$
(c) $K - Q = 0$
(d) $P(Q)\cdot Q - s\cdot K - w\cdot L = 0$

From (a) we find:

(a) $r = r + (MP(K) + \lambda 1 + \lambda 2\cdot(s - r))/(\lambda 2)$ where $(MP(K) + \lambda 1 + \lambda 2\cdot(s - r))/(\lambda 2) >0$

Again, instead of an incentive to over-invest in capital, pursuit of a minimization of sales objective function leads one to under-invest in capital since the relative cost of capital is less than the marginal value of the product from capital. The incentive is even higher with the inclusion of λ1 in the solution.

9. Min Costs (View of Public or Large Users)

For this case, we are interested to see the impact on investment decisions should one try to minimize costs as an objective. Here, the problem can be stated as:

Min $r\cdot K + w\cdot L$

s.t. $P(Q)\cdot Q - s\cdot K - w\cdot L = 0$

The Lagrangian is:

Min $L = r\cdot K + w\cdot L - \lambda 2\cdot(P(Q)\cdot Q - s\cdot K - w\cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = r - \lambda 2\cdot(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial K) + \lambda 2\cdot s = 0$

$LL = w - \lambda 2\cdot(P + Q\cdot(\partial P(Q)/\partial Q))\cdot (\partial Q/\partial L) + \lambda 2\cdot w = 0$

$L\lambda 2 = P(Q)\cdot Q - s\cdot K - w\cdot L = 0$

Performing substitutions and rearranging terms produces:

(a) $r - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$
(b) $w - \lambda 2 \cdot w + \lambda 2 \cdot w = 0$
(c) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a) we find:

(a) $r = r + (r + \lambda 2 \cdot (s - r))/(\lambda 2)$ where $(r + \lambda 2 \cdot (s - r))/(\lambda 2) > 0$

Instead of an incentive to over-invest in capital, pursuit of a minimization of costs objective function leads one to under-invest in capital since the relative cost of capital is less than the marginal value of the product from capital. This is the same results as found when trying to maximize VOS.

10. Min Costs with a Capacity Constraint (View of Public or Large Users)

For this case, we are interested to see the impact on investment decisions should one try to minimize costs as an objective while having adequate capacity to meet demand. Here, the problem can be stated as:

Min $r \cdot K + w \cdot L$

s.t. $Q <= K$
$P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

The Lagrangian is:

Min $L = r \cdot K + w \cdot L - \lambda 1 \cdot (Q - K) - \lambda 2 \cdot (P(Q) \cdot Q - s \cdot K - w \cdot L)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LK = r + \lambda 1 - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial K) + \lambda 2 \cdot s = 0$

$LL = w - \lambda 2 \cdot (P + Q \cdot (\partial P(Q)/\partial Q)) \cdot (\partial Q/\partial L) + \lambda 2 \cdot w = 0$

$L\lambda 1 = K - Q = 0$

$L\lambda 2 = P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

Performing substitutions and rearranging terms produces:

(a) $r + \lambda 1 - \lambda 2 \cdot r + \lambda 2 \cdot s = 0$
(b) $w - \lambda 2 \cdot w + \lambda 2 \cdot w = 0$
(c) $K - Q = 0$
(d) $P(Q) \cdot Q - s \cdot K - w \cdot L = 0$

From (a) we find:

(a) $r = r + (r + \lambda 1 + \lambda 2 \cdot (s - r))/(\lambda 2)$ where $(r + \lambda 1 + \lambda 2 \cdot (s - r))/(\lambda 2) > 0$

Again, instead of an incentive to over-invest in capital, pursuit of a minimization of costs objective function leads one to under-invest in capital since the relative cost of capital is less than the marginal value of the product from capital. The incentive is even higher with the inclusion of λ1 in the solution. This is also the same results found under maximization of the VOS.

Appendix B: Pricing Model

Assume:

Q1 = off-peak load
P1 = off-peak price
Q2 = peak load
P2 = peak price
C(Q1) = operating cost during off-peak periods
MCQ1 = off-peak marginal operating cost
C(Q2) = operating cost during peak periods
MCQ2 = peak marginal operating cost
K = capacity or capital required for a level of capacity
r = cost of capital (including depreciation)
λ1, λ2 = Lagrange multipliers

This analysis examines the outcomes from the point of view of different objective functions.

1. Maximize Welfare (View of a Utility Commission)

Define consumer value of service (VOS) as:

$$VOS = \int_{0}^{Q1(0)} f(P1)dQ1 + \int_{0}^{Q2(0)} f(P2)dQ2$$

Where P(Q1) and P(Q2) are the inverse demand functions.

Problem becomes:

Max Welfare (Consumer and Producer Surplus) =

$$\int_0^{Q1(0)} f(P1)dQ1 + \int_0^{Q2(0)} f(P2)dQ2 - C(Q1) - C(Q2) - r \cdot K$$

s.t. $Q2 <= K$
$P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Lagrangian problem is:

Max $W = VOS - C(Q1) - C(Q2) - r \cdot K + \lambda 1(K-Q2) + \lambda 2(P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K)$

Then, taking derivatives, Kuhn-Tucker equations become:

$WQ1 = P1 - MCQ1 + \lambda 2 \cdot P1 - \lambda 2 \cdot MCQ1 = 0$

$WQ2 = P2 - MCQ2 - \lambda 1 + \lambda 2 \cdot P2 - \lambda 2 \cdot MCQ2 = 0$

$WK = -r + \lambda 1 - r \cdot \lambda 2 = 0$

$W \lambda 1 = K - Q2 = 0$

$W\lambda 2 = P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Rearranging terms produces these conditions:

(a) $P1 = MCQ1$
(b) $P2 = MCQ2 + (\lambda 1/(1+\lambda 2))$
(c) $K = Q2$
(d) $r = (\lambda 1/(1+\lambda 2)$

which makes $P2 = MCQ2 + r$ or marginal operating cost plus marginal cost of capital.

These are the typical standard conclusions for welfare maximization.

2. Profit Maximization (View of a Utility)

Problem becomes:

Max Profit $(P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K)$

s.t. $Q2 <= K$
$P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Lagrangian problem is:

Max $L = P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K + \lambda 1(K-Q2) + \lambda 2(P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LQ1 = P1 - MCQ1 + \lambda 2 \cdot P1 - \lambda 2 \cdot MCQ1 = 0$

$LQ2 = P2 - MCQ2 - \lambda 1 + \lambda 2 \cdot P2 - \lambda 2 \cdot MCQ2 = 0$

$LK = -r + \lambda 1 - r \cdot \lambda 2 = 0$

$L \lambda 1 = K - Q2 = 0$

$L\lambda 2 = P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Rearranging terms produces these conditions:

(a) $P1 = MCQ1$
(b) $P2 = MCQ2 + (\lambda 1/(1+\lambda 2))$
(c) $K = Q2$
(d) $r = (\lambda 1/(1+\lambda 2)$

which makes $P2 = MCQ2 + r$ or marginal operating cost plus marginal cost of capital.

These are the same conclusions as found for welfare maximization.

3. Maximize Value of Service (VOS) (View of the Consumer)

Problem becomes:

Max VOS

s.t. $Q2 <= K$
$P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Lagrangian problem is:

$\text{Max } V = VOS + \lambda 1(K-Q2) + \lambda 2(P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K)$

Then, taking derivatives, Kuhn-Tucker equations become:

$VQ1 = P1 + \lambda 2 \cdot P1 - \lambda 2 \cdot MCQ1 = 0$

$VQ2 = P2 - \lambda 1 + \lambda 2 \cdot P2 - \lambda 2 \cdot MCQ2 = 0$

$VK = \lambda 1 - r \cdot \lambda 2 = 0$

$V \lambda 1 = K - Q2 = 0$

$V\lambda 2 = P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Rearranging terms produces these conditions:

(a) $P1 = MCQ1 \cdot (\lambda 2/(1+\lambda 2))$
(b) $P2 = (\lambda 1/(1+\lambda 2)) + (\lambda 2/(1+\lambda 2)) \cdot MCQ2$
(c) $K = Q2$
(d) $r = \lambda 1/\lambda 2$ or $\lambda 2 = \lambda 1/r$

Substituting $\lambda 2 = \lambda 1/r$ for $\lambda 2$ in (a) and (b) above and rearranging terms produces:

(a) $P1 = MCQ1 \cdot (\lambda 1/(r+\lambda 1))$
(b) $P2 = (MCQ2 + r) \cdot (\lambda 1/(r+\lambda 1))$

Thus, if both r and $\lambda 1 > 0$, this implies that under this approach, off-peak and peak prices would be set below marginal cost. Further, given the condition that $- r \cdot \lambda 2 = 0$, if $\lambda 2>0$, it implies that the cost of capital would be 0. So, under the VOS maximization objective, prices would be set below marginal cost which would encourage consumers to purchase above an economically efficient level of service.

4. Minimize Sales (View of Environmental Advocates)

Problem becomes:

$$\text{Min } Q1 + Q2$$

s.t. $Q2 <= K$
$P1 \cdot Q1 + P2 \cdot Q2 – C(Q1) – C(Q2) – r \cdot K = 0$

Lagrangian problem is:

$$\text{Min } L = Q1 + Q2 + \lambda 1(K-Q2) + \lambda 2(P1 \cdot Q1 + P2 \cdot Q2 – C(Q1) – C(Q2) – r \cdot K)$$

Then, taking derivatives, Kuhn-Tucker equations become:

$$LQ1 = 1 + \lambda 2 \cdot P1 - \lambda 2 \cdot MCQ1 = 0$$

$$LQ2 = 1 - \lambda 1 + \lambda 2 \cdot P2 - \lambda 2 \cdot MCQ2 = 0$$

$$LK = \lambda 1 - r \cdot \lambda 2 =0$$

$$L \lambda 1 = K – Q2 =0$$

$$L\lambda 2 = P1 \cdot Q1 + P2 \cdot Q2 – C(Q1) – C(Q2) – r \cdot K =0$$

Rearranging terms produces these conditions:

(a) $P1 = MCQ1 – 1/ \cdot \lambda 2$
(b) $P2 = MCQ2 + ((\lambda 1-1)/\lambda 2) = MCQ2 + \lambda 1/ \lambda 2 -1/\lambda 2$
(c) $K = Q2$
(d) $r = \lambda 1/\lambda 2$ or $\lambda 2 = \lambda 1/r$

Note: If $r = \lambda 1/\lambda 2$ and $\lambda 2 = \lambda 1/r$, the pricing equations become:

(a) $P1 = MCQ1 – r/\lambda 1$
(b) $P2 = MCQ2 + r – r/\lambda 1$

For both the off-peak and peak prices, if $\lambda 1>0$, it is apparent that the prices would be below marginal costs by $r/\lambda 1$

5. Minimize Costs (View of Interveners in rate cases – large users and consumer advocates)

Problem becomes:

Min $C(Q1) + C(Q2) + r \cdot K$

s.t. $Q2 <= K$

$P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Lagrangian problem is:

Min $L = C(Q1) + C(Q2) + r \cdot K + \lambda 1(K-Q2) + \lambda 2(P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K)$

Then, taking derivatives, Kuhn-Tucker equations become:

$LQ1 = MCQ1 + \lambda 2 \cdot P1 - \lambda 2 \cdot MCQ1 = 0$

$LQ2 = MCQ2 - \lambda 1 + \lambda 2 \cdot P2 - \lambda 2 \cdot MCQ2 = 0$

$LK = r + \lambda 1 - r \cdot \lambda 2 = 0$

$L \lambda 1 = K - Q2 = 0$

$L\lambda 2 = P1 \cdot Q1 + P2 \cdot Q2 - C(Q1) - C(Q2) - r \cdot K = 0$

Rearranging terms produces these conditions:

(a) $P1 = MCQ1 \cdot (\lambda 2 - 1)/\lambda 2$
(b) $P2 = MCQ2 \cdot (\lambda 2 - 1)/\lambda 2 + \lambda 1/\lambda 2$
(c) $K = Q2$
(d) $r = -\lambda 1/(1-\lambda 2)$ or $r = \lambda 1/(\lambda 2-1)$ or $\lambda 2 = (r + \lambda 1)/r$

Note: If $\lambda 2 = (r + \lambda 1)/r$, the pricing equations become:

(c) $P1 = MCQ1 \cdot (\lambda 1/(r + \lambda 1))$
(d) $P2 = MCQ2 \cdot (\lambda 1/(r + \lambda 1)) + \lambda 1/\lambda 2 = (MCQ2 + r) \cdot (\lambda 1/(r + \lambda 1))$

For this cost minimization approach, if r and $\lambda 1 > 0$, again it becomes apparent that the prices would be less than the marginal costs.

Summary Comparison

Case	Off-Peak Price	Peak Price
Max Welfare	MCQ1	MCQ2 + r
Max Profit	MCQ1	MCQ2 + r
Max VOS	$MCQ1 \cdot (\lambda 1/(r+\lambda 1))$	$(MCQ2 + r) \cdot (\lambda 1/(r+\lambda 1))$
Min Sales	$MCQ1 - r/\lambda 1$	$MCQ2 + r - r/\lambda 1$

Min Costs	$MCQ1 \cdot (\lambda 1/(r + \lambda 1))$	$(MCQ2 + r) \cdot (\lambda 1/(r + \lambda 1))$